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Electric dipole moment searches: Effect of linear electric field frequency shifts induced in confined gases, II

R. Golub

Physics Dept, NCSU

C.M. Swank

Physics Dept

North Carolina State Universty,

Riddick Hall and

Raleigh, NC 27610

S.K. Lamoreaux

Yale University

Department of Physics

PO Box 208120 and

New Haven, Conn. 06520

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Abstract

The next generation of particle edm searches will be at such a high sensitivity that it will be possible for the results to be contaminated by a systematic error resulting from the interaction of the motional $(E \times v/c)$ magnetic field with stray field gradients. In this paper we extend previous work to present an analytic form for the frequency shift in the case of a rectangular storage vessel and discuss the implications of the result for the neutron edm experiment which will be installed at the SNS (Spallation Neutron Source) by the LANL collaboration

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I. INTRODUCTION

Searches for particle electric dipole moments (edm) are considered to be one of the most promising places to search for physics beyond the standard model. Current experiments have reached the sensitivity where they have to take into account a systematic effect due to the influence on the particle's magnetic dipole moment of a combination of the motional magnetic field, $\left(\overrightarrow{B}_m = \overrightarrow{E} \times \overrightarrow{v}/c\right)$ due to the motion of the particle in the applied static electric field, and gradients in the ambient magnetic field. For slow motion (adiabatic limit) the effect can be described as a geometric phase effect [1] whereas a description in terms of the Bloch-Siegert shift is valid for fast motions as well ([2]). A treatment valid also for intermediate motions for particles moving in a cylindrical container has been given in [3], [4]. In this note we discuss some additional symmetries of the effect and present an analytic solution for the case of particles moving in a rectangular vessel.

II. SYMMETRY OF THE EFFECT

It is easy to see [2], that the effect depends only on motion in a plane perpendicular to the direction of $\overrightarrow{E}$ (parallel motion produces no $\overrightarrow{B}_m$). Equations (23 and 36) in [3] show that in general the frequency shift linear in $\overrightarrow{E}$ which leads to the systematic error is given in terms of the spectrum of the velocity autocorrelation function by

$$\Delta\Omega_E = -\frac{\gamma^2 E}{2c}\left[\frac{\partial B_y}{\partial y}S_y(\omega_o) + \frac{\partial B_x}{\partial x}S_x(\omega_o)\right] \tag{1}$$

with

$$S_i(\omega_o) = \int_0^\infty \cos\omega_o\tau R_i(\tau)\,d\tau \tag{2}$$

$$R_i(\tau) = 2\int_0^\tau \psi_i(x)\,dx \tag{3}$$

$$\psi_i(x) = \langle v_i(t)\,v_i(t-x)\rangle \tag{4}$$

$$S_i(\omega_o) = 2\int_{-\infty}^{\infty} \frac{\Psi_i(\omega)}{(\omega_0^2 - \omega^2)}d\omega \tag{5}$$

where $\omega_o = \gamma B_o$ is the Larmor frequency, γ is the gyromagnetic ratio, B_o is the homogeneous magnetic field taken as parallel to $\overrightarrow{E}$ in the z direction and $\Psi_i(\omega)$ is defined by

$$\psi_i(x) = \int_{-\infty}^{\infty} \cos\omega x\Psi_i(\omega)\,d\omega \tag{6}$$

. If the field is cylindrically symmetric

$$\frac{\partial B_y}{\partial y} = \frac{\partial B_x}{\partial x} = -\frac{1}{2}\frac{\partial B_z}{\partial z} \tag{7}$$

$$\Delta\Omega_E = \frac{\gamma^2 E}{2c}\frac{1}{2}\frac{\partial B_z}{\partial z}\left(S_y(\omega_o) + S_x(\omega_o)\right) \tag{8}$$

and the result depends only on $\frac{\partial B_z}{\partial z}$ for all frequencies and geometries of the orbits. In the case that the trajectories have cylindrical symmetry

$$S_y(\omega_o) = S_x(\omega_o) = S(\omega_o) \tag{9}$$

$$\Delta\Omega_E = \frac{\gamma^2 E}{2c}\frac{\partial B_z}{\partial z}S(\omega_o) \tag{10}$$

This symmetry will hold in the high frequency limit ($\omega_o >> \omega_r$) which is determined by the short time behavior of the correlation function, and the result that the shift in this case depends only on $\frac{\partial B_z}{\partial z}$ has been obtained in [2], section IV B and by a different method in [4] section IV A. The advantage of the present treatment is obvious.

An interesting case that can arise in practice is the symmetry

$$\frac{\partial B_z}{\partial z} \approx 0, \quad \frac{\partial B_x}{\partial x} \approx -\frac{\partial B_y}{\partial y} >> 0 \tag{11}$$

$$\Delta\Omega_E = \frac{\partial B_x}{\partial x} \left(S_x \left(\omega_o \right) - S_y \left(\omega_o \right) \right) \tag{12}$$

In the high frequency limit this difference in the spectra will approach zero.

III. MOTION IN A RECTANGULAR BOX

A. Single velocity

In a rectangular box with specular reflecting walls the orbits are straight lines reflecting at equal angles to the normal when they encounter a wall. With perpendicular walls parallel to the x and y axes the motion in each dimension will be independent of the other dimension and the angle with the normal will be preserved independently for the walls along x and y. At a wall collision the magnitude of velocity and the component parallel to the wall are unchanged and the perpendicular component of velocity changes sign. Thus the correlation function for a given velcocity component ($i = x, y$) starts at v_i^2 and after l collisions is

$$\psi_i \left(\tau \right) \equiv \left\langle v_i \left(\tau \right) v_i \left(0 \right) \right\rangle = v_i^2 \left(-1 \right)^l \quad for \quad lT_i < \tau < \left(l+1 \right) T_i \tag{13}$$

with the time between collisions $T_i = L_i / v_i$, being constant for each orbit and L_i the length of the box in direction i.

Thus the velocity correlation function for each direction is a square wave with the switching points being equally spaced but whose exact timing depends on the distance of the starting point of a given orbit from the first wall collision. Averaging over these starting points proceeds as in equations 14-17 of [4]. In fact the orbits are exactly those characterized by $\alpha = \pi/2$ in that reference (see fig.2 [4]) and the results of that paper can be applied to the present case by substituting

$$\alpha = \pi/2 \tag{14}$$

$$R = L_i/2 \tag{15}$$

Thus equ. 42 [4] becomes

$$S_i \left(\omega_o \right) = \frac{L_i^2}{4} \sum_{m=-\infty}^{\infty} \frac{1}{\left(\pi \left(m + 1/2 \right) \right)^2} \left[\frac{\left(\omega_o''^2 - \left(\pi \left(m + 1/2 \right) \right)^2 \right)}{\left(\left(\omega_o''^2 - \left(\pi \left(m + 1/2 \right) \right)^2 \right)^2 + \omega_o''^2 r_o^2 \right)} \right] \tag{16}$$

with $\omega_o'' = \omega_o L_i/2v_i$ and $r_o = L_i/2\lambda$, λ being the mean free path between gas collisions. Introducing $\omega_o' = \omega_o L_i/v_i$ we have

$$S_i\left(\omega_o, l_o\right) = L_i^2 \sum_{m=-\infty}^{\infty} \frac{1}{\left(\pi\left(m+1/2\right)\right)^2} \left[\frac{\left(\omega_o'^2 - \left(\pi\left(2m+1\right)\right)^2\right)}{\left(\left(\omega_o'^2 - \left(\pi\left(2m+1\right)\right)^2\right)^2 + \omega_o'^2 l_o^2\right)}\right] \tag{17}$$

with $l_o = \frac{L_i}{\lambda}$. This is plotted in figure 1), which is to be compared with figure 3) of [4] for the case of a cylinder.

Equation (17) and figure 1) apply to motion in one dimension. For the full 2 dimensional problem it is necessary to add together suitably normalized forms of the function for each dimension.

The value of $S_i\left(\omega_o = 0\right) = .083L_i^2$ agrees with that predicted by the diffusion theory for the single dimension contribution to a rectangular box (eqation 82, [3]),

$$\frac{8}{\pi^4} \sum_{m=1,3,5} \frac{1}{m^4} = .083 \tag{18}$$

Equation 17 shows that $S_i\left(\omega_o\right)$ becomes independent of L_i as the frequency increases and the larger the damping (larger l_o), the higher the frequency where this occurs.

B. Frequency shift averaged over Maxwell velocity distribution, the case of co-magnetometers.

Both the neutron edm experiment carried out by Baker *et al,* [5] and that being developed by the Los Alamos collaboration [6], make use of co-magnetometers, that is a gas of atoms occupying the same region as the ultra-cold neutrons and satisfying the Maxwell-Bolzmann velocity distribution. In this case we write the velocity as

$$v_i = y\beta(T) \tag{19}$$

with $\beta\left(T\right) = \sqrt{\frac{2kT}{m}}$ the most probable velocity in a volume. Then our one dimensional velocity v_i has the probability distribution

$$P\left(y\right)dy = \frac{2}{\sqrt{\pi}} e^{-y^2} dy \tag{20}$$

The spectral function of the frequency shift can then be rewritten

$$S_i\left(\omega_o, y, l_o^*\right) = L_i^2 \sum_{m=-\infty}^{\infty} \frac{1}{\left(\pi\left(m+1/2\right)\right)^2} \left[\frac{\left(\omega_o^{*2} - \left(\pi\left(2m+1\right)\right)^2 y^2\right) y^2}{\left(\left(\omega_o^{*2} - \left(\pi\left(2m+1\right)\right)^2 y^2\right)^2 + \omega_o^{*2}\left(l_o^*\right)^2\right)}\right] \tag{21}$$

where $\omega_o^* = \omega_o L_i/\beta(T)$, and $l_o^* = L_i/(\beta(T)\tau_c(T))$ and we have specialized to the case where the gas collision time, τ_c is independent of velocity. This is valid for the common case where the scattering cross section satisfies $\sigma \sim 1/v$ and holds in particular for the case of He^3 diffusing in superfluid He^4 which is the co-magnetometer in the LANL experiment [6], [7].

1. *He^3 colliding with phonons in superfluid He^4, the co-magnetometer in the LANL search for a neutron electric dipole moment.*

Since the velocity of the He^3 is much less than the phonon velocity, the collision rate of the phonons with the He^3 will be independent of velocity and the mean free path satisfies $\lambda = v\tau_c$. We obtain τ_c from

$$\tau_c(T) = 3D(T)/\left\langle v^2\right\rangle_T \tag{22}$$

with $\langle v^2\rangle_T$ the mean square velocity in a volume of gas and $D(T)$ has been measured at temperatures of interest [8]

$$D(T) = \frac{1.6}{T^7} cm^2/\sec \tag{23}$$

We now average the frequency shift over the Maxwell- Boltzman distribution for velocity in one dimension

$$\Psi_x(\omega_o^*, T) = -\frac{2}{L_i^2\sqrt{\pi}} \int e^{-y^2} S_i(\omega_o^*, y, l_o^*)\, dy \tag{24}$$

The results are plotted in figure 2):

The same result is shown as a function of temperature for fixed (normalized) frequency in fgure 3. (Note the frequency normalization is temperature dependent.

C. A rectangular box with the two sides significantly different,

Equations 17 and 24 refer to a single dimension. If the second dimension of the box has a length L_y, then equation 4 can be written $(\varepsilon = L_y/L_x)$, keeping the same normalization for S_y and ω_o^*:

$$\frac{S_y(\omega_o, y, l_o^*)}{L_x^2} = \varepsilon^2 \sum_{m=-\infty}^{\infty} \frac{1}{(\pi(m+1/2))^2} \left[\frac{\left(\omega_o^{*2} - (\pi(2m+1))^2 y^2\right) y^2}{\left(\left(\varepsilon^2\omega_o^{*2} - (\pi(2m+1))^2 y^2\right)^2 + \omega_o^{*2}(l_o^*)^2 \varepsilon^2\right)} \right] \tag{25}$$

Averaging this as in equation 24 we obtain, $(\Psi_y(\omega_o^*, T, l_o^*) = \Psi_x(\varepsilon\omega_o^*, T, l_o^*\varepsilon))$

$$\Psi(\omega_o^*, T) = \Psi_x(\omega_o^*, T) + \varepsilon^2\Psi_y(\omega_o^*, T) \tag{26}$$

Figure 4) shows the two dimensional result for $\varepsilon = 0.2$, normalized to L_x^2 as a function of frequency normalized to L_x for various temperatures while figure 5) shows the normalized shift vs temperature for various normalized frequencies.

In figure 6) we show the contribution to the frequency shift for the two directions independently, compared to the results of numerical simulations, as well as the total frequency shift given by their sum for T=0.4K. The contribution of the short side (y) has been normalized to the long side (x) for the case $\varepsilon = L_y/L_x = 0.2$. We are assuming, for this discussion that the two components of the gradients are equal

$$\frac{\partial B_o}{\partial x} = \frac{\partial B_o}{\partial y} = \frac{1}{2}\frac{\partial B_o}{\partial z} \tag{27}$$

Otherwise each curve in fig. 6 will have to be multiplied by the appropriate gradient and the total effect will be altered.

We see that around the zero crossing the contributions of the two dimensions contribute with opposite sign so the symmetry (equation 9) does not hold and if we wish to operate near the zero crossing the result will depend on $\frac{\partial B_x}{\partial x}$ and $\frac{\partial B_y}{\partial y}$ separately and not on $\frac{\partial B_z}{\partial z}$ except in the case of cylindrical symmetry $\frac{\partial B_o}{\partial x} = \frac{\partial B_o}{\partial y} = \frac{1}{2}\frac{\partial B_o}{\partial z}$.

Finally fig.7 shows the comparison of theory and numerical simulation for T=300mK.

IV. DISCUSSION

The calculations of the velocity correlation function (vcf) presented here and in [4], start by following a single trajectory with the collisions only damping the amplitude of the vcf and not changing the velocity components, v_x, v_y as the particle moves along this trajectory. The simulations, on the other hand follow a particle as it is deflected to another (randomly chosen) trajectory by the collisions. Essentially the theory follows the particles that haven't collided while the simulations follow those that have.

Each collision, while conserving the kinetic energy (magnitude of velocity), will result in a change of direction of the motion and hence of the x and y components of velocity so the correlation function resulting from the simulation of a single trajectory will be quite different

from that considered by the theoretical calculation. However, for each collision that takes a particle from trajectory (1) to trajectory (2) there should be a collision leading to the reverse transition according to detailed balance and when we average over all trajectories the results are seen to be the same.

In addition while we have neglected the motion parallel to the E field we see that collisions will alter the velocity component in this direction, and, as a result of conservation of energy in the collisions, will thus alter the velocity in the perpendicular plane. However this effect will be cancelled when we average over all trajectories in the perpendicular plane as we have done here and in [4]. (We note that due to the heavy mass and slow He^3 velocity, Baym and Ebner [9] conclude that the phonon scattering on He^3 is predominantly elastic.)

We have presented the general solution for the frequency shift linear in E, for the case of a rectangular box with specular reflecting walls. The effects of non-specular wall reflections are expected to be small for the case of heavy damping of interest with respect to the co-magnetometers. For UCN the non-specular wall collisions are expected to be the major source of damping but the effect is expected to be small. This will be discussed in a subsequent work.

V. FIGURE CAPTIONS

Fig. 1) Normalized frequency shift vs normalized frequency, $\omega' = \omega_o L_x/v$, for the single dimension contribution of a single velocity.

Fig. 2) One dimensional contribution to the normalized velocity averaged frequency shift vs. reduced frequency $\omega_x = \omega_o L_x/\beta(T)$, for Temperatures T=0.1, 0.2 0.3 and 0.4K, using the temperature dependent mean free path for He^3 in He^4

Fig. 3) One dimensional contribution to normalized velocity averaged frequency shift with normalized frequency ($\omega^* = \omega L_i/\beta(T)$) as a parameter.

Fig. 4) Normalized frequency shift for a rectangular box with $\varepsilon = L_y/L_x = 0.2$ normalized to L_x^2 vs frequency normalized to $L_x/\beta\,(T)$ for various temperatures.

Fig. 5) Frequency shift in a rectangular box with $\varepsilon = L_y/L_x - 0.2$ normalized to L_x^2 vs temperature, K, with normalized frequency as a parameter.

Fig. 6) Contribution of the long dimension, x, (red),short dimension, y, (blue) and

combned result (violet) for the frequency shift normalized to long dimension (x),vs, frequency normalized to the longdimension, $\omega 1 = \omega^* = \omega_o L_x/\beta(T)$ for a temperature of 0.4K and $\varepsilon = L_y/L_x = 0.2$ The results of the theory are shown along with those derived by numerical simulations.

Fig. 7) Contribution of the long dimension, x, (green),short dimension, y, (red) and combned result (violet) for the frequency shift normalized to long dimension (x),vs, frequency normalized to the longdimension, $\omega^* = \omega_o L_x/\beta(T)$ for a temperature of 0.3K and $\varepsilon = L_y/L_x = 0.2$ The results of the theory are shown along with those derived by numerical simulations.

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